2.3-2.5 Derivative Rules: Form B Key

Tutor copy. Every answer was checked by symbolic computation from the formulas as printed (SymPy, `verify_form.py`). Provenance: `PROVENANCE_FORMB.md`. To the extent possible under law, the author(s) have waived all copyright and related or neighboring rights to this key. CC0 1.0 Universal: `https://creativecommons.org/publicdomain/zero/1.0/`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit.
- **Any valid method.** Expanding first, negative exponents, the quotient rule, simplifying first, solving for y first where possible, or carrying the rules all the way through are all accepted; no method is required.
- **Notation.** y', $\dfrac{dy}{dx}$, $f'(x)$ and $\dfrac{d}{dx}[\ ]$ are all accepted.
- **Value vs. derivative.** A function value where a derivative was asked (or a first derivative where a second was asked) earns 0 for that part.
- **Simplifying.** Answers need not be simplified; equivalent forms are correct.
- **Excluded values.** Where a formula fails at some values, the key may name them. No points are deducted for omitting statements of excluded values.
- **Trap notes** print the line that a skipped step leaves out, next to the answer the skip produces.

Question 3 columns run x, $f'(x)$, $g(x)$, $f''(x)$, $g'(x)$, $f(x)$.

Form B

Item	Answer	Work, rubric, trap
1(a)	$3x^2 \cot x$ $-(x^3+2)\csc^2 x$	Product rule: $3x^2\cot x + (x^3+2)(-\csc^2 x)$. *Rubric:* 1 pt each product-rule term; 1 pt both signs. *Trap:* $\dfrac{d}{dx}[\cot x] = -\csc^2 x$; the line to write is $(x^3+2)\cdot(-\csc^2 x)$. $+(x^3+2)\csc^2 x$ is the slip.
1(b)	$\dfrac{x^2-6x-1}{(x^2+1)^2}$	$\dfrac{(-1)(x^2+1)-(3-x)(2x)}{(x^2+1)^2} = \dfrac{-x^2-1-6x+2x^2}{(x^2+1)^2}$. *Rubric:* 2 pts quotient rule in the right order; 1 pt answer (unsimplified is fine). *Trap:* $(3-x)(2x) = 6x-2x^2$, and the minus reaches both terms: the line to write is $-(6x-2x^2) = -6x+2x^2$.
1(c)	0	No x appears: $y = \frac{2}{2} = 1$, a constant. *Rubric:* 3 pts, all or nothing. *Trap:* The quotient rule here treats fixed angles as x.
1(d)	$6x^{-5/2} - 12x$	$y = 8x^{-1/2} - 2x^3$; $y' = -4x^{-3/2} - 6x^2$; $y'' = 6x^{-5/2} - 12x$. *Rubric:* 1 pt rewriting with exponents; 1 pt y'; 1 pt y''. *Trap:* $(-4)\left(-\frac{3}{2}\right) = +6$; $-6x^{-5/2}$ is the dropped sign. Stopping at y' answers a different question (0 pts).
2(a)	$-\sin x$	Every four derivatives return $\sin x$; $42 = 4\cdot 10+2$, so the answer is the second derivative: $\cos x$, $-\sin x$. *Rubric:* 2 pts the cycle of four; 1 pt $42 = 40+2$; 2 pts answer. *Trap:* $\cos x$ is the first derivative, not the 42nd. The 42 counts derivatives; $\sin^{42} x$ would be a power.

Item	Answer	Work, rubric, trap
2(b)	-3 (at $t=2$)	$v=3t^2-12t+9$; $a=6t-12=0$ at $t=2$; $v(2)=12-24+9=-3$. *Rubric:* 1 pt v; 1 pt a; 1 pt $t=2$; 2 pts -3. *Trap:* Speed 3 drops the sign; $s(2)=3$ is the position; $t=2$ is the time, not the velocity.
3(a)	$\frac{7}{25}$	$q'=\dfrac{f'g-fg'}{g^2}=\dfrac{(2)(5)-(-3)(-1)}{5^2}=\dfrac{10-3}{25}$. *Rubric:* 2 pts quotient rule with f on top; 1 pt reads; 2 pts value. *Trap:* Reversing the numerator gives $-\frac{7}{25}$. $(-3)(-1)=+3$, so the line is $10-3$; $10+3$ is the slip.
3(b)	-16	$r'=2f\,f'$; $r''=2f'\,f'+2f\,f''=2(f')^2+2f\,f''$; at 2: $2(4)+2(-3)(4)=8-24$. *Rubric:* 2 pts r'' (product rule on $2ff'$); 1 pt reads; 2 pts value. *Trap:* $2f(2)f''(2)=-24$ drops the $2(f')^2$ term; $2f'(2)f''(2)=16$ differentiates f twice in one factor.
4(a)	$-\dfrac{\csc(\sqrt{x})\cot(\sqrt{x})}{2\sqrt{x}}$	$-\csc(\sqrt{x})\cot(\sqrt{x})\cdot\frac{1}{2}x^{-1/2}$. *Rubric:* 1 pt outer $-\csc\cot$; 1 pt inner $\frac{1}{2}x^{-1/2}$; 1 pt answer. *Trap:* Dropping the inner $\frac{1}{2\sqrt{x}}$, or the minus of $\dfrac{d}{dx}[\csc x]$.
4(b)	$-\dfrac{8x}{(2x^2+1)^{3/2}}$	$f=4(2x^2+1)^{-1/2}$; $f'=-2(2x^2+1)^{-3/2}\cdot 4x$. *Rubric:* 1 pt exponent $-\frac{3}{2}$; 1 pt inner $4x$; 1 pt answer. *Trap:* $-\frac{1}{2}-1=-\frac{3}{2}$; writing $-\frac{1}{2}+1=\frac{1}{2}$ is the slip. The quotient rule also works.
4(c)	$8t\sin^3(t^2)\cos(t^2)$	$\sin^4(t^2)=(\sin(t^2))^4$: $4\sin^3(t^2)\cdot\cos(t^2)\cdot 2t$. *Rubric:* 1 pt power layer; 1 pt $\cos(t^2)$; 1 pt inner $2t$. *Trap:* $\sin^4(t^2)$ is a fourth power of a sine, not $\sin(t^8)$ or $\sin(4t^2)$.
4(d)	$-\sin(x)(1-3x)^4$ $-12\cos(x)(1-3x)^3$	Product: $(-\sin x)(1-3x)^4+\cos x\cdot 4(1-3x)^3(-3)$. *Rubric:* 1 pt product rule; 1 pt $-\sin x$; 1 pt $4(1-3x)^3(-3)$. *Trap:* The inner -3 makes the second term negative; $+12\cos x(1-3x)^3$ is the slip.
5(a)	3	$g(3)=-3$, $g'(3)=-1$ (segment from $(1,-1)$ to $(4,-4)$); $f'(-3)=-3$ (segment from $(-4,4)$ to $(-2,-2)$); $k'(3)=(-3)(-1)$. *Rubric:* 1 pt $g(3)$, $g'(3)$; 2 pts f' at -3; 1 pt value. *Trap:* $f'(3)\,g'(3)=(1)(-1)=-1$ reads f' at 3; the sign flips.
5(b)	2	$f(2)=-3$, $f'(2)=1$; $g'(-3)=2$ (segment from $(-4,-3)$ to $(-2,1)$); $m'(2)=(2)(1)$. *Rubric:* 1 pt $f(2)$, $f'(2)$; 2 pts g' at -3; 1 pt value. *Trap:* $g'(2)\,f'(2)=(-1)(1)=-1$ reads g' at 2.
6	$y=-12(x-2)$	$f(2)=1^3-1=0$; $f'(x)=3(5-x^2)^2(-2x)$, $f'(2)=-12$; $y-0=-12(x-2)$. *Rubric:* 2 pts $f(2)=0$; 2 pts $f'(2)$; 2 pts line. *Trap:* The point is $(2,0)$ on the x-axis, so the line starts $y-0$. $f(2)=1$ forgets the -1; $+12$ loses the inner $-2x$.
7(a)	1	$f(g(x))=\left(\sqrt[3]{x-2}\right)^3+2=x$, so $h'(x)=1$. The chain rule also gives $f'(g(10))\,g'(10)=f'(3)\cdot\frac{1}{12}=12\cdot\frac{1}{12}$. *Rubric:* 3 pts, all or nothing for 1; either route. *Trap:* $f'(10)\,g'(10)$ reads f' at the wrong input.
7(b)	0	$(\sin u+\cos u)^2=1+2\sin u\cos u$, so the bracket is 1. *Rubric:* 3 pts, all or nothing for 0; either route. *Trap:* The grind works but only gives 0 after every term cancels.
8(a)	$\dfrac{2+\cos y}{2y+x\sin y}$	$2yy'-[\cos y+x(-\sin y)\,y']=2$; $2yy'-\cos y+x\sin y\,y'=2$. *Rubric:* 1 pt $2yy'$; 1 pt product rule on $x\cos y$; 1 pt $-\sin y\,y'$; 1 pt solving. *Trap:* Two minus signs meet in $-x\cdot(-\sin y\,y')=+x\sin y\,y'$; $2y-x\sin y$ in the denominator is the slip. The $2x$ differentiates to 2.
8(b)	$\dfrac{1-2y}{2x}$	Expand: $8xy=4x+8$, so $8y+8xy'=4$. The direct route also works. *Rubric:* 1 pt expansion or chain rule on both squares; 1 pt product rule on xy; 1 pt right side 4; 1 pt solving. *Trap:* $-(2x-y)^2=-4x^2+4xy-y^2$; the minus reaches all three terms.
9(a)	1	$y'+g'(xy)\,(y+xy')=0$. At $(2,-1)$, $xy=-2$: $g'(-2)=-1$. $y'-(-1+2y')=0$, so $y'=1$. *Rubric:* 1 pt chain rule on $g(xy)$; 1 pt product rule on xy; 1 pt g' read at -2; 2 pts value. *Trap:* g' is read at the input $xy=-2$, not at $x=2$ (slope $\frac{1}{2}$ gives $\frac{1}{4}$). Check: $-1+g(-2)=-1+2=1$.

Item	Answer	Work, rubric, trap
9(b)	$y+1=x-2$	$y-(-1)=1\cdot(x-2)$. *Rubric:* 4 pts; follow-through from (a). *Trap:* $y-(-1)=y+1$; $y-1$ is the slip.
10	$y=3(x-2)$, $y=\frac{3}{2}(x+4)$	On the x-axis $y=0$: $x^2+2x-8=0$, $x=2$ or $x=-4$. $y'=\dfrac{y-2x-2}{2y-x}$: 3 at $(2,0)$, $\frac{3}{2}$ at $(-4,0)$. *Rubric:* 2 pts both points; 2 pts y'; 1 pt each slope; 1 pt both lines. *Trap:* Crossing the x-axis means $y=0$; setting $x=0$ finds the y-intercepts. At $(-4,0)$: $y-0=\frac{3}{2}(x-(-4))=\frac{3}{2}(x+4)$.
11	$\dfrac{3y^2-x^2}{2xy+\sec^2 x}$	With respect to y: $2x\dfrac{dx}{dy}\,y+x^2+\sec^2 x\dfrac{dx}{dy}=3y^2$. *Rubric:* 1 pt product rule on x^2y with respect to y; 1 pt $\sec^2 x\dfrac{dx}{dy}$; 2 pts answer. *Trap:* Differentiating with respect to x gives $\dfrac{dy}{dx}=\dfrac{2xy+\sec^2 x}{3y^2-x^2}$, the reciprocal (0 pts).
12	-14	$y'=\dfrac{2x}{y}=4$; $y''=\dfrac{2y-2xy'}{y^2}=\dfrac{2-4\cdot 4}{1}$. *Rubric:* 2 pts y'; 4 pts quotient rule with y' inside; 2 pts value. *Trap:* Substituting $(2,1)$ into y' first gives 4, whose derivative is 0. $-2xy'=-16$; losing that sign gives 18.

Points: 100.